29. Suppose $A_1, A_2, ..., A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, ..., B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B,'S. then n is equal to

(A) 15 (B) 3 (C) 45 (D) 35

$\therefore$, $n(S) = \frac{30 \times 5}{10}$

$n(S) = 15$ (P.T.O)

$A_i = \{ e_{i1}\ e_{i2}\ e_{i3}\ e_{i4}\ e_{i5} \}$, $1 \le i \le 30$

$B_j = \{ k_{j1}\ k_{j2}\ k_{j3} \}$, $1 \le j \le n$

$\forall i \cup A_i$

$M_A =$

e_{11}	e_{12}	β	e_{14}	e_{15}
e_{21}	e_{22}	α		
		β		
		β		
		α		
$e_{30,1}$	$e_{30,2}$		$e_{30,4}$	$e_{30,5}$

10 copies — 10 times — 30×5

$30 \times 5 = 10 \times n(S)$

$M_B =$

k_{11}	β	α_{12}	k_{13}
α_{21}	β		α
k_{n1}	α	β k_{n2}	k_{n3}

$n \times 3$

$n \times 3 = 9 \times n(S)$

$S = \{ \ldots\ \alpha\ \ldots\ \beta \}$

$n(S) = \frac{n \times 3}{9} = \frac{30 \times 5}{10} \Rightarrow n = 45$ $n(S)$

29. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B,'S. then n is equal to

(A) 15 (B) 3 (C) 45 (D) 35

$n = 45$

$n(S) = 15$

Mathematics XI
Set Counting Problem from NCERT Exemplar

Avijit

October 1, 2025

Justification for the answer that the union set S has 15 elements (α_i). $S = \{\alpha_1, \ldots, \alpha_{15}\}$

So, matrix M_A (size 150) will have to contain 15 unique elements each repeated 10 times. Now, one column of M_A has 30 entries. So I can fit three unique elements (30 blank entries filled with 10 repeated copies = 3 unique per column) in one single column of M_A.

Now M_A has 5 columns, so, M_A can utmost take $5 \times 3 = 15$ unique entries, each being written 10 times. This specific arrangement is shown below.

However, the arrangement of M_A does not matter as it does not alter the number of elements. There are over 10^{120} different possible arrangements of M_A, out of which we have chosen only one specific, easy-to-visualize arrangement.

Similar analysis can be done for the matrix M_B.

$$M_A = \begin{bmatrix}
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\alpha_1 & \alpha_4 & \alpha_7 & \alpha_{10} & \alpha_{13} \\
\hline
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\alpha_2 & \alpha_5 & \alpha_8 & \alpha_{11} & \alpha_{14} \\
\hline
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15} \\
\alpha_3 & \alpha_6 & \alpha_9 & \alpha_{12} & \alpha_{15}
\end{bmatrix}$$

10 copied of α_1

10 repeated

10 repeated

10 repeated